# cKanren

## miniKanren with Constraints

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# Overview

1. Introduction to Logic Programming/miniKanren
2. Introduction to Constraints
3. Examples
4. Implementation Overview

# miniKanren

Logic programming language extending Scheme

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Three important operators: $\equiv$, *fresh*, and $cond^e$

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Intuition:
The *goal* `(== 5 5)` *succeeds* while `(== 5 6)` *fails*

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Three important operators: $\equiv$, *fresh*, and *cond*$^e$

Intuition:
The *goal* `(== 5 5)` *succeeds* while `(== 5 6)` *fails*

```
(fresh (x)
  (conde
    ((== x 5))
    ((== x 6))))
```

*unifies* $x$ with 5 *or* 6

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Logic programming language extending Scheme

Uses *run* as an interface operator

```
(run1 (y)
  (fresh (x z)
    (== x z)
    (== 3 y)))
```

$\Rightarrow$ `(3)`

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```
(run1 (y)
  (fresh (y)
    (conde
      ((== y 4))
      ((== y 5))))
  (== 3 y))
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$\Rightarrow$ (3)

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Imposing a certain restriction on a variable or set of variables
Find a solution such that every constraint is satisfied

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Examples: Set of equations

$$\begin{aligned} x + y + z &= h \\ h + 3 &= m - x \\ y - 7 &= h + z \end{aligned}$$

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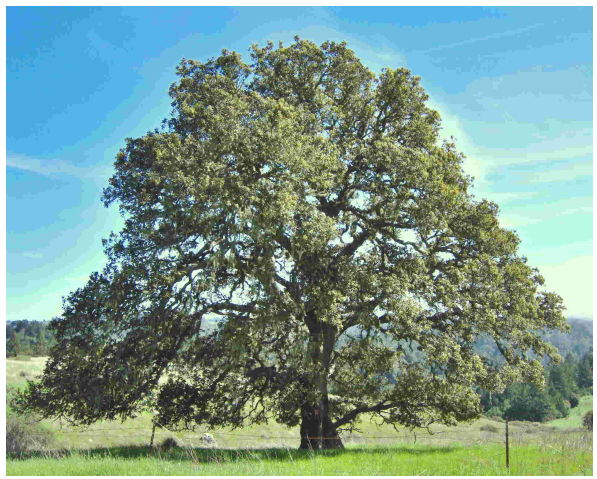

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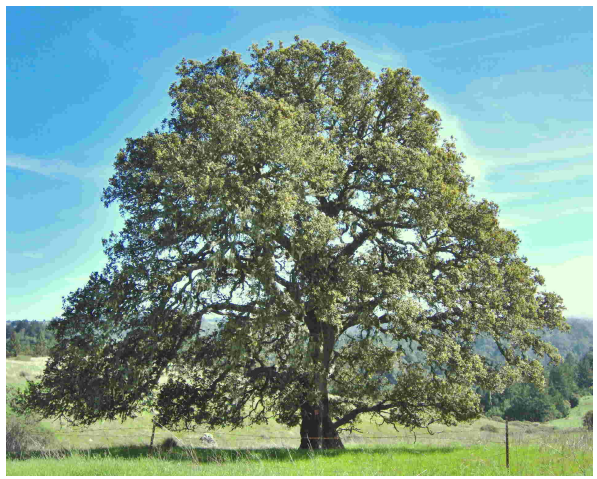

$$\text{'oak} \not\equiv \text{'pine}$$

$$\text{'}((1\ x)\ y\ 7) \not\equiv \text{'}(z\ 5\ w)$$

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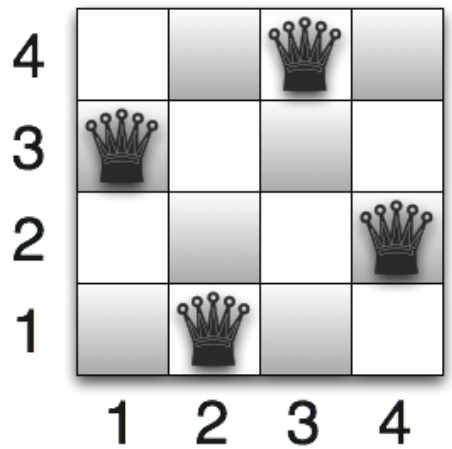
Examples: Set of equations, Tree Disequality, N-Queens



# Send More Money

Find the correct letter values to satisfy the following equation:

$$\begin{array}{r} S\ E\ N\ D \\ +\quad M\ O\ R\ E \\ \hline M\ O\ N\ E\ Y \end{array}$$

Each letter represents a different digit in the range 0 through 9

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- Performs very slowly on standard constraint problems
- Extensions to miniKanren are incompatible with each other

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- A framework for defining constraint systems on top of miniKanren
- Retains all miniKanren functionality
- Includes two constraint systems:
  *finite domains* and *tree disequality*
- Easy to add or compose additional constraints systems

# Constraints Over Finite Domains

We can associate a *domain* with a variable $x$

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We consider only *finite domains* of natural numbers
such as $x \in \{1, 2, 3, 7, 8, 9\}$

... but there are others (interval domains, boolean domains, etc.)

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Derived goals:

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- $(all\text{-}diff^{fd}\ v*)$

Derived goals:

- $in^{fd}$ to assign multiple variables a single initial domain
- $(<^{fd} u\ v)$

# Example

```
(run* (q)
  (fresh (x y z)
    (domfd x '(7 8 9 10))
    (domfd y '(4 5 8 9 12))
    (domfd z '(1 2 12 16))
    ...))
```

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$x \in \{7, 8, 9, 10\}$

$y \in \{4, 5, 8, 9, 12\}$

$z \in \{1, 2, 12, 16\}$

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    (<=fd x y)
    (+fd x y z)
    (=/=fd x y)
    ...))
```

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    (+fd x y z)
    (=/=fd x y)
    (== q `(,x ,y ,z))))
```

$\Rightarrow$ `((7 9 16))`

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  (fresh (x y z)
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    (=/=fd x y)
    (== q `(,x ,y ,z))
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New operator $\not\equiv$ (more general than $\not\equiv^{fd}$)

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(run* (q)
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$\Rightarrow$ `((1 1) (2 2) (1 2) (2 1))`

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    (=/= `(,x ,y) `(,y ,x))
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```

$\Rightarrow$ `((1 2) (2 1))`

# Implementation Overview

# Data Structures

cKanren uses a package to store information

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Substitution
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Example: $((x\ .\ (7\ 8\ 9))\ (y\ .\ (2\ 3\ 4\ 5)))$

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Substitution
Example: $((x\ .\ 1)\ (y\ .\ \#\mathrm{t})\ (z\ .\ x))$

Domain store
Example: $((x\ .\ (7\ 8\ 9))\ (y\ .\ (2\ 3\ 4\ 5)))$

Constraint store
Example: $((\texttt{proc}\ \leq^{fd}\ y\ x)\ (\texttt{proc}\ \textit{all-diff}^{fd}\ '(x\ z\ h\ 7)))$

# Framework

1. $\equiv$
2. Fixpoint algorithm
3. Consistency checks
4. *reify*

# Equivalence

$\equiv$

- Only constraint that is not kept in the constraint store

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$\equiv$

- Only constraint that is not kept in the constraint store
- Uses miniKanren unification

# Fixpoint Algorithm

No constraints directly interact with one another

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A framework function reruns constraints on newly ground variables

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A framework function reruns constraints on newly ground variables

Example:

```
(run* (q)
  (fresh (x)
    (infd x q '(1 2 3))
    (+fd x 1 q)
    ...
    (== x 2)
    ...))
```

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   For example, $x$ from previous slide, after being unified with 2

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   For example, $x$ from previous slide, after being unified with 2
2. Grabs current constraint store
   Constraint store `(...  (proc +fd x 1 q) ...)`
3. Run every constraint involving any variables in $x*$ again
   ... but only if the constraint is still in the store

   Reruns $+^{fd}$ constraint with new information that $x$ is 2.

# Consistency

Programs with irrelevant but unsatisfiable constraints will fail

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```
(run* (q)
  (fresh (x y z)
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    (all-difffd `(,x ,y ,z))
    (== q 5)))
```

$\Rightarrow$ ()

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(run* (q)
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    (== q 5)))
```

$\Rightarrow$ ()

Before returning anything to the user, each variable with finite domain constraints is re-evaluated, to guarantee that there is *at least one* acceptable value for each constrained variable.

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*reify*

- Produces the final result returned to the user

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```
(run* (q) (=/= q 5))
```

$\Rightarrow$ `(( _0 : (=/= ((_0 . 5))))) `

# Parameters

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Consistency checks before reification

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*enforce-constraints*
Consistency checks before reification

*reify-constraints*
Builds a Scheme data structure that packages the constraint information in a way that is readable to the user

# Composition

Having multiple constraint systems in the same session is tricky, as parameter definitions will overwrite each other

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```
(let ((ls (run* (q) (n-queens q 8))))
  (run* (s) (all-diffo ls)))
```

Implementor must define parameters in a way that makes sense

# Future Work

- Performance
- Specialized operators
- Adding $\alpha$Kanren
- Using different domains? Simulaneously?

# Questions?